Principio di Induzione e Disuguaglianze Notevoli

1. $\displaystyle\sum_{i=1}^{n}(2i-1)=n^2 \quad \forall n\in\mathbb{N}, n\geq 1$

2. $\displaystyle\sum_{i=0}^{n} i=\frac{n(n+1)}{2} \quad \forall n\in\mathbb{N}$

3. $\displaystyle\sum_{i=0}^{n} i^2=\frac{n(n+1)(2n+1)}{6} \quad \forall n\in\mathbb{N}$

4. $\displaystyle\sum_{i=0}^{n} i^3=\left(\frac{n(n+1)}{2}\right)^2 \quad \forall n\in\mathbb{N}$

5. $\displaystyle\sum_{i=1}^{n} \frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1} \quad \forall n\in\mathbb{N}, n\geq 1$

6. $\displaystyle\binom{n}{i}+\binom{n}{i+1}=\binom{n+1}{i+1} \quad \forall n\in\mathbb{N}, n\geq 1,$ per ogni $i=1,\ldots,n-1$

7. $\displaystyle\sum_{i=0}^{n}\binom{n}{i}=2^n=(1+1)^n \quad \forall n\in\mathbb{N}$

8. $\displaystyle\sum_{i=0}^{n}\binom{n}{i}a^{n-i}b^i=(a+b)^n \quad \forall n\in\mathbb{N}$

9. $\displaystyle\sum_{i=0}^{n}\binom{n}{i}^2=\binom{2n}{n} \quad \forall n\in\mathbb{N}$

10. $\displaystyle\sum_{i=0}^{n}(-1)^i\binom{n}{i}=0 \quad \forall n\in\mathbb{N}, n>0$

11. Disuguaglianza di Bernoulli: $(1+a)^n\geq 1+an$ per ogni $a>-1$ e $n\in\mathbb{N}$.

12. $2^n>n \quad \forall n\in\mathbb{N}$

13. Per quali $n\in\mathbb{N}$ vale $2^n>n+1$?

14. Trovare il minimo numero naturale n_0 tale che $n^2-6n+8\geq 0$ per ogni $n\in\mathbb{N}, n\geq n_0$.

15. Sia $n_1=\min\{n\in\mathbb{N}\mid(1+x)^n>1+nx+nx^2 \quad \forall x>0\}$. Determinare n_1 e l'insieme $\{n\in\mathbb{N}\mid(1+x)^n>1+nx+nx^2 \quad \forall x>0\}$.